ÉRETTSÉGI VIZSGA • 2020. május 5.

MATEMATIKA
ANGOL NYELVEN

EMELT SZINTŰ
ÍRÁSBELI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI
ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

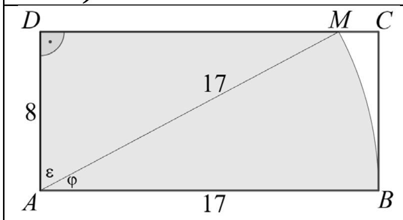
1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect**, enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

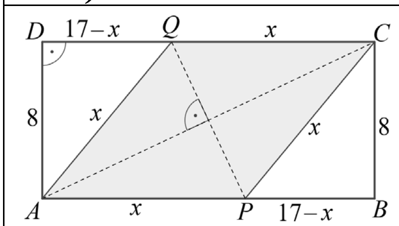
Assessment of content:

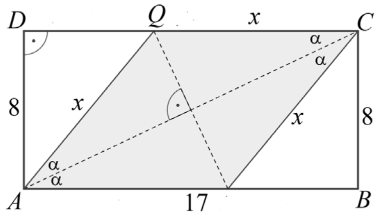
1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error**, no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.

6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
11. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
12. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
13. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
14. **Assess only four out of the five problems in part II of this paper**. The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

1. a)		
 Diagram reflecting correct understanding of the problem.	1 point	
(Using the notations of the diagram) from the right triangle *ADM*: $DM = \sqrt{17^2 - 8^2} = 15$ (cm).	1 point	
The area of triangle *ADM*: $\frac{8 \cdot 15}{2} = 60$ (cm^2).	1 point	
From triangle *ADM*: $\cos\varepsilon = \frac{8}{17}$, $\varepsilon \approx 61.93°$.	1 point	
The central angle of sector *BAM*: $\varphi = 90° - \varepsilon \approx 28.07°$,	1 point	
the area of the sector: $\frac{17^2\pi \cdot 28.07°}{360°} \approx 70.8$ (cm^2).	1 point	
The area of the covered part of the rectangle is approximately 60 + 70.8 = 130.8 cm^2.	1 point	
Total:	**7 points**	

1. b) Solution 1		
 Correct diagram. (The length of the side of the rhombus is *x* cm.)	1 point	
In the right triangle *AQD* (apply the Pythagorean Theorem): $(17-x)^2 + 8^2 = x^2$.	1 point	
$289 - 34x + x^2 + 64 = x^2$	1 point	
$34x = 353$ $x = \frac{353}{34} \approx 10.38$ (cm)	1 point	
The perimeter of the rhombus is $4x = \frac{706}{17} \approx 41.5$ cm.	1 point	
Total:	**5 points**	

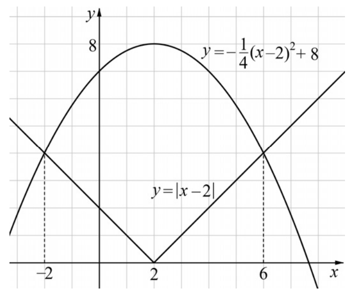
1. b) **Solution 2**		
 Correct diagram. (The length of the side of the rhombus is x cm.)	1 point	
If the angle $CAB\angle = \alpha$, then in the right triangle ABC $\tan\alpha = \frac{8}{17}$, $\alpha \approx 25.2°$.	1 point	
$DAQ\angle = 90° - 2\alpha \approx 39.6°$	1 point*	
From triangle ADQ: $x = \frac{8}{\cos 39.6°} \approx 10.38$ (cm).	1 point*	
The perimeter of the rhombus is $4x \approx 41.5$ cm.	1 point	
Total:	**5 points**	

*Note: The 2 points marked by * may also be given for the following reasoning.*

$AC = \sqrt{17^2 + 8^2} = \sqrt{353}$ (≈ 18.79) (cm)	1 point	
(Use the Law of Sines in triangle ACQ) $\frac{x}{\sqrt{353}} = \frac{\sin 25.2°}{\sin(180° - 2 \cdot 25.2°)}$, and so $x \approx 10.38$ (cm).	1 point	

2. a) **Solution 1**		
If $x > 2$ then the original equation is: $x - 2 = 7 + x - 0.25x^2$;	1 point	
If $x \le 2$ then the original equation is $2 - x = 7 + x - 0.25x^2$.	1 point	
Rearranging to zero the first case gives $x^2 - 36 = 0$, while the second case gives $x^2 - 8x - 20 = 0$.	1 point	
The roots of the equation $x^2 - 36 = 0$ are 6 and –6,	1 point	
the roots of $x^2 - 8x - 20 = 0$ are 10 and –2.	1 point	
The roots –6 and 10 are not part of the domain (6 and –2 are).	1 point	*These 2 points are also due if the candidate gives the correct answer by checking all 4 solutions by substitution.*
Only equivalent steps were taken, so 6 and –2 are correct solutions of the original equation.	1 point	
Total:	**7 points**	

Note: Award a maximum of 3 points if the candidate ignores the absolute values, solves the equation $x - 2 = 7 + x - 0.25x^2$ in the set of real numbers and checks the solutions.

2. a) Solution 2		
$7 + x - 0.25x^2 = -\frac{1}{4}(x-2)^2 + 8$	1 point	
Diagram of the function f: $x \mapsto \lvert x-2 \rvert$.	1 point	
Diagram of the function g: $x \mapsto -\frac{1}{4}(x-2)^2 + 8$ (in the same coordinate system).	2 points	
The first coordinates of the points of intersection are: $x_1 = -2$ and $x_2 = 6$.	1 point	
$f(-2) = g(-2) = 4$ and $f(6) = g(6) = 4$, so there are two solutions –2 and 6.	2 points	
Total:	**7 points**	

2. a) Solution 3		
$7 + x - 0.25x^2 = -\frac{1}{4}(x-2)^2 + 8$	1 point	
$(x-2)^2 = \lvert x-2 \rvert^2$, so $\lvert x-2 \rvert = -\frac{1}{4}\lvert x-2 \rvert^2 + 8$.	1 point	
The equation given is quadratic for $\lvert x-2 \rvert$: $\lvert x-2 \rvert^2 + 4\lvert x-2 \rvert - 32 = 0$.	1 point	
The solutions are $\lvert x-2 \rvert = 4$ or $\lvert x-2 \rvert = -8$.	1 point	
In the first case $x_1 = -2$ or $x_2 = 6$,	1 point	
the second case is not possible (as $\lvert x-2 \rvert \geq 0$).	1 point	
Check by substitution or reference to equivalent steps.	1 point	
Total:	**7 points**	

2. b) Solution 1		
Domain: $x^2 - 200 > 0$ (i.e. $x^2 > 200$).	1 point	
The base 2 logarithm function is strictly increasing and so, from the original inequality $x^2 - 200 < 2^{20}$, that is $x^2 < 2^{20} + 200$ follows.	1 point	
$200 < x^2 < 2^{20} + 200$, so $\sqrt{200} < \lvert x \rvert < \sqrt{2^{20} + 200}$ (≈ 1024.1).	1 point	
(As x is an integer) $15 \leq \lvert x \rvert \leq 1024$.	1 point	
The number of the possible values of $\lvert x \rvert$ is 1024 – 14 (=1010).	1 point	
As x may also be a negative integer, the number of solutions of the inequality is $2 \cdot (1024 - 14) = 2020$.	1 point	
Total:	**6 points**	

2. b) Solution 2		
Domain: $x^2 - 200 > 0$ (i.e. $x^2 > 200$).	1 point	
The base 2 logarithm function is strictly increasing and so, from the original inequality $x^2 - 200 < 2^{20}$, that is $x^2 < 2^{20} + 200$ follows.	1 point	
There are 14 positive square numbers between 1 and 200 ($14^2 = 196$).	1 point	*The smallest possible value of x^2 is 15^2,*
As $(2^{10})^2 < 2^{20} + 200 < (2^{10} + 1)^2$, the largest possible value of x^2 is $(2^{10})^2$.	1 point	$2^{20} + 200 = 1\,048\,776$, $(2^{10}+1)^2 = 1\,050\,625$
The number of the possible values of x^2 is $2^{10} - 14$ (=1010).	1 point	
As x may also be a negative integer, the number of solutions of the inequality is $2\cdot(2^{10} - 14) = 2020$.	1 point	
Total:	**6 points**	

3. a)		
If 26.0% of the number of those seeking work is 67 000 then 1% is $\left(\frac{67\,000}{26} \approx\right) 2577$,	1 point	The number of people looking for work is $\frac{67\,000}{0.26} \approx 257\,692$,
and the total number of people looking for work is about 257 700.	1 point	
Rounded to the nearest ten thousand this is 260 thousand people.	1 point	*Do not award this point if the solution is not rounded or rounded incorrectly.*
Total:	**3 points**	

Note: As 26.0% is a rounded value, the following are true for the number u of people looking for work: $\frac{67\,000}{0.2605} < u \le \frac{67\,000}{0.2595}$*, that is* $257\,197 \le u \le 258\,189$.
If the 67 000 people, reported in the news, is a number rounded to the nearest thousand then it can actually be any element of the set $[66\,500; 67\,499] \cap \mathbf{N}$ *and so* $255\,279 \le u \le 260\,112$.
In this case 260 000 *people is still the correct answer.*

3. b) Solution 1		
Express the number of people looking for work (u) in terms of the working-age population *(w people)*: $u = w \cdot 0.038$. Express it in terms of the active population (a): $u = a \cdot 0.056$.	1 point	
$w \cdot 0.038 = a \cdot 0.056$	1 point	
$a = w\cdot\frac{0.038}{0.056}\left(= w\cdot\frac{19}{28}\right) \approx w \cdot 0.679$.	1 point	
The active population is about 68% of the working-age population.	1 point	
Total:	**4 points**	

Note: Award a maximum of 2 points if the candidate gives their answer using the ratio of 3.8 and 5.6 (or 0.038 and 0.056) without any further explanation.

3. b) **Solution 2**		
(The number of people looking for work is 260 000, based on part a) of the question, so) the active population is 260 000 : 0.056 ≈ 4 640 000 people,	1 point	
the working-age population is 260 000 : 0.038 ≈ 6 840 000 people.	1 point	
The ratio of these numbers is (4 640 000 : 6 840 000 ≈) 0.678.	1 point	
The active population is about 68% of the working-age population.	1 point	
Total:	**4 points**	

Note: Award full score if the candidate miscalculated the number of people looking work in part a) but uses this value otherwise correctly and gives the correct answer in part b).

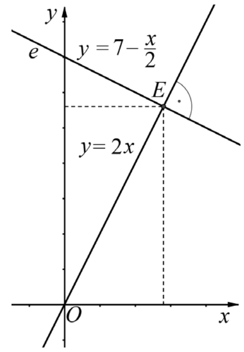
3. c)		
For example 1, 2, 3, 4, 5, 6, 21.	2 points	
The median of this example is 4, the mean is $\left(\frac{1+2+3+4+5+6+21}{7}=\frac{42}{7}=\right)6$.	1 point	
Total:	**3 points**	

Note: Award a maximum of 2 points if the candidate's list contains the same number(s) more than once.

3. d) **Solution 1**		
The median shows the (minimal) maximal salary of half of all workers. The mean salary does not give any information about the actual distribution of the salaries. (The median may be more or less than the mean.)	2 points	
Total:	**2 points**	

3. d) **Solution 2**		
If a (relatively small) group of the workers earn exceptionally high salaries (i.e. much higher than the median) then the mean salary may be higher than the median. This explains how Mr. Virág's salary is more than that of the lower half of the workers, and yet, it is below the mean salary.	2 points	
Total:	**2 points**	

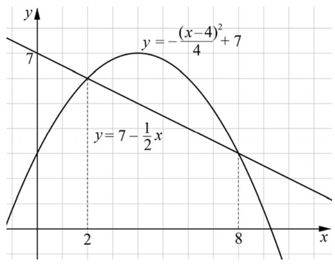
Note: The candidate may also refer to their solution in part c). For example, the seven numbers given above in part c) have a median of 4 and a mean of 6. In this case Mr. Virág's salary would be 5: it is greater than half of the data, yet "below average".

4. a)		
(Line e does not cross the origin.) The gradient of the other diagonal of the square is 2; this line does cross the origin and its equation is $y = 2x$.	1 point	
The centre E of the square is the point of intersection of the lines of the diagonals. The coordinates of this point are given by the solution of the system: $\left.\begin{array}{l} y=7-\frac{1}{2}x \\ y=2x \end{array}\right\}$	1 point	
(Use substitution:) $2x=7-\frac{1}{2}x$, $x=\frac{14}{5}=2.8$, $y=2\cdot 2.8=5.6$. Therefore $E(2.8; 5.6)$.	2 points	
The distance of the origin and point E: $OE=\sqrt{2.8^2+5.6^2}=\sqrt{39.2}$.	1 point*	
The area of the square is $\frac{2OE\cdot 2OE}{2}=$	1 point*	*The side of the square is* $a=\sqrt{2}\cdot OE=\sqrt{2}\cdot\sqrt{39.2}$,
= 78.4 (area units).	1 point	*the area is* $a^2=78.4$ *(area units).*
Total:	**7 points**	

*Note: The 2 points marked by * may also be given for the following reasoning:*

The equation of line e may be rearranged into the form $x + 2y - 14 = 0$. (Use the formula for the distance between a point and a line) the distance between the origin and the line e is $\left\lvert\frac{0+2\cdot 0-14}{\sqrt{5}}\right\rvert=\frac{14}{\sqrt{5}}$	1 point	
The area of the square is $\left(\frac{14}{\sqrt{5}}\right)^2\cdot 2=$	1 point	$\frac{\left(\frac{28}{\sqrt{5}}\right)^2}{2}=$

Note:
The coordinates of the vertices of the square are (0; 0), (8.4; 2.8), (5.6; 11.2), (–2.8; 8.4).

4. b)		
The solutions of the equation $7-\frac{1}{2}x=-\frac{(x-4)^2}{4}+7$ will give the first coordinates of the points of intersection of the two graphs.	1 point	
$x^2-10x+16=0$,	1 point	
$x_1=2$, $x_2=8$.	1 point	
Let f: $x\mapsto 7-\frac{1}{2}x$ and g: $x\mapsto -\frac{(x-4)^2}{4}+7$ ($x\in\mathbf{R}$). (As the graph of function g is above the graph of function f between the points of intersection) the area in question is: $A=\int_2^8(g-f)$.	1 point*	$A=\left\lvert\int_2^8(f-g)\right\rvert$
$g(x)-f(x)=-\frac{(x-4)^2}{4}+7-7+\frac{1}{2}x=\frac{-x^2+10x-16}{4}$	1 point*	
$T=\int_2^8\frac{-x^2+10x-16}{4}\mathrm{d}x=\left[-\frac{x^3}{12}+\frac{5x^2}{4}-4x\right]_2^8=$	1 point*	
$=-\frac{8^3}{12}+\frac{5\cdot 8^2}{4}-4\cdot 8-\left(-\frac{2^3}{12}+\frac{5\cdot 2^2}{4}-4\cdot 2\right)=$ $=\frac{16}{3}+\frac{11}{3}=9$ (area units)	1 point*	
Total:	**7 points**	

*Note: The 4 points marked by * may also be given for the following reasoning:*

(Between the points of intersection the graph of the parabola is above the x axis, so) the area below the graph of the parabola is $\int_2^8\left(-\frac{(x-4)^2}{4}+7\right)\mathrm{d}x=\int_2^8\left(-\frac{1}{4}x^2+2x+3\right)\mathrm{d}x=$ $=\left[-\frac{x^3}{12}+x^2+3x\right]_2^8=$	1 point	$\int_2^8\left(-\frac{(x-4)^2}{4}+7\right)\mathrm{d}x=$ $=\left[-\frac{(x-4)^3}{12}+7x\right]_2^8=$
$=-\frac{8^3}{12}+8^2+3\cdot 8-\left(-\frac{2^3}{12}+2^2+3\cdot 2\right)=\frac{136}{3}-\frac{28}{3}=36$.	1 point	$=\frac{152}{3}-\frac{44}{3}=36$
A right trapezium is located below the segment that connects the two points of intersection (and above the x axis). The lengths of the parallel sides of this trapezium are ($f(2)=$) 6 and ($f(8)=$) 3, the height of the trapezium is 6, the area is $\frac{6+3}{2}\cdot 6=27$.	1 point	
The area in question is 36 – 27 = 9 (area units).	1 point	

II.

5. a) **Solution 1**		
If the first term of the sequence is 1, there are 10 possibilities: 1, 3, 5, 7; 1, 3, 5, 8; 1, 3, 5, 9; 1, 3, 6, 8; 1, 3, 6, 9; 1, 3, 7, 9; 1, 4, 6, 8; 1, 4, 6, 9; 1, 4, 7, 9; 1, 5, 7, 9.	2 points*	
If the first term of the sequence is 2, there are four possibilities: 2, 4, 6, 8; 2, 4, 6, 9; 2, 4, 7, 9; 2, 5, 7, 9.	1 point	
If the first term of the sequence is 3, there is only one possibility: 3, 5, 7, 9.	1 point	
(The first term of the sequence may not be greater than 3.) There are a total (10 + 4 + 1 =) 15 possible ways to select the first four terms of the sequence.	1 point	
Total:	**5 points**	

*Note: The following are considered as errors: there are consecutive numbers among the four terms listed, there are less than 10 possible solutions given, there are 10 possible solutions given but some of these are identical. Award 1 out of the 2 points marked by * if the candidate's solution has one or two errors, 0 points if there are more than 2 errors.*

5. a) **Solution 2**		
Arrange the element of the set {1;2; 3; 4; 5; 6; 7; 8; 9} in increasing order and mark each selected element with ● and all non-selected elements with ×. (For example, ● × ● × × ● × ● × means that the numbers 1, 3, 6, 8 are selected.)	1 point	
Each suitable set of four numbers can be uniquely matched to a string of 9 symbols that contains 5 ×-s and 4 ●-s such that there are no adjacent ●-s.	1 point	
Setting the 5 ×-s produces 6 possible places to set the 4 ●-s (* × * × * × * × * × *).	1 point*	
There are a total $\binom{6}{4}$ = 15 possible ways to select the first four terms of the sequence.	2 points*	
Total:	**5 points**	

*Note: The 3 points marked by * may also be given for the following reasoning:*

Place four ●-s and separate them with three ×-s: ● × ● × ● × ●. The remaining two ×-s may be placed into any two of the 5 slots determined by the ●-s, including repeats.	1 point	
This is a case of combination with repeat, 2 out of 5, i.e. there are $\binom{5+2-1}{2}=\binom{6}{2}=15$ possibilities.	2 points	

5. a) **Solution 3**		
Subtract 1 from the second term, 2 from the third, 3 from the fourth. This way you will obtain the first four terms of a strictly increasing sequence where each term is an element of the set $\{1; 2, 3, 4, 5, 6\}$.	2 points	
The other way round: if four elements of the set $\{1; 2; 3; 4; 5; 6\}$ are selected, arranged in increasing order, 1 is added to the second number, 2 to the third and 3 to the fourth then the 4 numbers obtained may be the first 4 terms of the required sequence.	1 point	
There are $\binom{6}{4} = 15$ ways to select these 4 terms. (As there is a one-to-one mapping between the selections and the sequences) there will be 15 different possible ways to select the first four terms of the sequence.	2 points	
Total:	**5 points**	

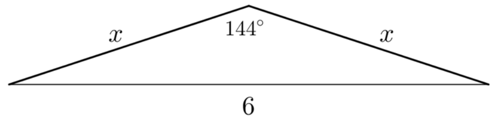
5. a) **Solution 4**		
Let $f(n, k)$ be the number of the strictly increasing sequences where the first k terms are selected from the set $\{1; 2; \ldots ; n\}$ and there are no consecutive integers among the terms of the sequence. Our task is now to determine $f(9, 4)$. The relation $f(n, k) = f(n-1, k) + f(n-2, k-1)$ is true as, given that n is not among the first k terms of the sequence, k elements will have to be selected from the set $\{1; 2; \ldots ; n-1\}$, while, given that n *is* a term of the sequence, $n-1$ may not be a term and so another $k-1$ elements must be selected from the set $\{1; 2; \ldots ; n-2\}$	2 points	

Initial conditions: $f(n, 1) = n$ and $f(7, 4) = f(5, 3) = f(3, 2) = f(1, 1) = 1$. Apply the recursive formula: $f(9, 4) = f(8, 4) + f(7, 3)$, where $f(8, 4) = f(7, 4) + f(6, 3) = 1 + f(6, 3)$ and $f(7, 3) = f(6, 3) + f(5, 2)$. $f(6, 3) = f(5, 3) + f(4, 2) = 1 + f(4, 2)$, $f(5, 2) = f(4, 2) + f(3, 1) = f(4, 2) + 3$, finally, $f(4, 2) = f(3, 2) + f(2, 1) = 1 + 2 = 3$. “Reverse engineering” the steps: $f(5, 2) = 6$, $f(6, 3) = 4, f(7, 3) = 10, f(8, 4) = 5, f(9, 4) = 15$.	3 points	$f(9, 4) =$ $= 1 + 2f(6, 3) + f(5, 2) =$ $= 6 + 3f(4, 2) =$ $= 6 + 3 \cdot 3 = 15$
Total:	**5 points**	

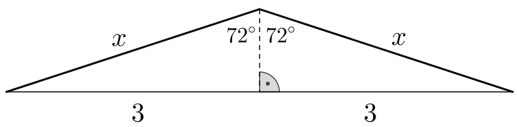
5. b)		
There are 900 three-digit numbers (total number of cases).	1 point	
Let a, b and c be consecutive digits ($b = a + 1$ and $c = b + 1 = a + 2$). In this case the solution may take the form $\overline{abc}$ or $\overline{cba}$.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
There are 7 numbers in $\overline{abc}$ form (as $1 \le a \le 7$), and 8 numbers in $\overline{cba}$ form (as $0 \le a \le 7$). This means a total of 15 suitable numbers (favourable cases).	2 points	$\overline{abc}$ *form*: 789, 678, 567, 456, 345, 234, 123; $\overline{cba}$ *form*: 210, 321, 432, 543, 654, 765, 872, 981. *This gives a total of 15 suitable numbers.*
The probability is $\frac{15}{900} = \frac{1}{60}$ (≈ 0.017).	1 point	
Total:	**5 points**	

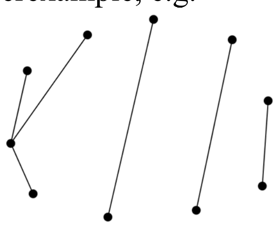
5. c)		
$b = a + 1$, $c = a + 2$ and $d = a + 3$. (Digits in base nine stand for 1, 9 and 81 and so) $N = 81a + 9(a + 1) + (a + 2)$; (digits in base eight stand for 1, 8 and 64 and so) $N = 64(a + 1) + 8(a + 2) + (a + 3)$.	2 points	
$81a + 9(a + 1) + (a + 2) = 64(a + 1) + 8(a + 2) + (a + 3)$	1 point	
$91a + 11 = 73a + 83$ $18a = 72$, i.e. $a = 4$.	1 point	
In base ten: $N = (91a + 11 =)$ 375.	1 point	
Check: $375_{10} = 456_9 = 567_8$, which are, in fact, equivalent in bases nine and eight.	1 point	
Total:	**6 points**	

Note: Award full score if the candidate gives the correct answer by checking the base nine form of each of the 5 possible numbers in base eight: ($123_8 = 83_{10} = 102_9$, $234_8 = 156_{10} = 183_9$, $345_8 = 229_{10} = 274_9$, $456_8 = 302_{10} = 365_9$, $567_8 = 375_{10} = 456_9$).

6. a)		
One interior angle of the regular decagon is 144°.	1 point	
 Let x be the length (in cm) of one side of the decagon. Apply the Law of Cosines: $6^2 = x^2 + x^2 - 2 \cdot x \cdot x \cdot \cos 144° = 2 \cdot (1 - \cos 144°) \cdot x^2$,	2 points*	
(as $x > 0$) $x \approx 3.15$ (cm).	1 point	
Total:	**4 points**	

*The 2 points marked by * may also be given for the following reasoning:*

 Given that the length of one side of the decagon is x, the right triangle shown yields: $\sin 72° = \frac{3}{x}$,	2 points	*The other two angles of the triangle cut off by the shortest diagonal are 18° each. Let x be the length of the side of the decagon (in cm). Use the Law of Sines:* $\frac{x}{6} = \frac{\sin 18°}{\sin 144°}$,

6. b)		
Not true.	1 point	
A correct counterexample, e.g. 	2 points	
Total:	**3 points**	

6. c) **Solution 1**		
An edge that does not share a vertex with the red edge is one of the edges of the remaining complete graph on ($n - 2$) vertices. There are $\binom{n-2}{2} = \frac{(n-2)(n-3)}{2}$ such edges.	2 points	
There are $n - 2$ edges joining the red edge in both endpoints, so there are $2(n - 2)$ edges that do share a vertex with the one first selected.	2 points	
As $P(A) = P(B)$ the total number of cases will be the same,	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*

and so the number of edges that do share a vertex with the red edge will be the same as the number of edges that do not share a vertex with the red edge: $\frac{(n-2)(n-3)}{2}=2(n-2)$.	2 points	
As $n-2\neq 0$, it is also true that $\frac{n-3}{2}=2$.	1 point	
$n=7$ (this is a graph on 7 vertices).	1 point	
Total:	**9 points**	

6. c) Solution 2		
A complete graph on n vertices has $\binom{n}{2}=\frac{n(n-1)}{2}$ edges. The second edge may therefore be selected from among $\frac{n(n-1)}{2}-1$ others (total number of cases).	1 point	
An edge that does not share a vertex with the red edge is one of the edges of the remaining complete graph on $(n-2)$ vertices. There are $\binom{n-2}{2}=\frac{(n-2)(n-3)}{2}$ such edges (number of favourable cases).	2 points	*There are $n-2$ edges joining the red edge in both endpoints, so there are $2(n-2)$ edges that do share a vertex with the red edge.*
As events A and B are complements and $P(A)=P(B)$, $P(A)=0.5$.	1 point	$P(B)=0.5$
$\frac{\frac{(n-2)(n-3)}{2}}{\frac{n(n-1)}{2}-1}=\frac{1}{2}$	1 point	$\frac{2(n-2)}{\frac{n(n-1)}{2}-1}=\frac{1}{2}$
$\frac{n^2-5n+6}{n^2-n-2}=\frac{1}{2}$	1 point	$\frac{4(n-2)}{n^2-n-2}=\frac{1}{2}$
Rearranged to zero: $n^2-9n+14=0$.	1 point	$\frac{4(n-2)}{(n-2)(n+1)}=\frac{1}{2}$
The solutions of the quadratic equation are 2 and 7.	1 point	(as $n\neq 2$) $\frac{4}{n+1}=\frac{1}{2}$ $8=n+1$,
(As $n\geq 3$) 2 is not a correct solution, the graph must have 7 vertices.	1 point	*$n=7$ (the graph has 7 vertices).*
Total:	**9 points**	

Note: Award 3 points if the candidate proves that the complete graph on 7 vertices is a correct solution but does not prove that there aren't any other solutions.

7. a) **Solution 1**		
Select first the four balls forming the second tier (the selection of the rest of the balls would not affect the probability anyway). The probability of the first ball being dark is $\frac{15}{30}$, that of the second is $\frac{14}{29}$, of the third is $\frac{13}{28}$ and of the fourth is $\frac{12}{27}$.	2 points	
The probability is the product of these four numbers: $\frac{15}{30}\cdot\frac{14}{29}\cdot\frac{13}{28}\cdot\frac{12}{27}$,	1 point	
that is $\frac{32\,760}{657\,720}=\frac{13}{261}\approx 0.0498$.	1 point	
Total:	**4 points**	

7. a) **Solution 2**		
There are $\binom{30}{4}$ (= 27 405) different ways to select, disregarding the order, the 4 balls forming the second tier (all of these are equally likely, this is the total number of cases).	1 point	
The number of favourable cases is: $\binom{15}{4}$(= 1365).	1 point	
The probability is the ratio of the above: $\frac{\binom{15}{4}}{\binom{30}{4}}$,	1 point	
that is $\frac{1365}{27\,405}=\frac{13}{261}\approx 0.0498$.	1 point	
Total:	**4 points**	

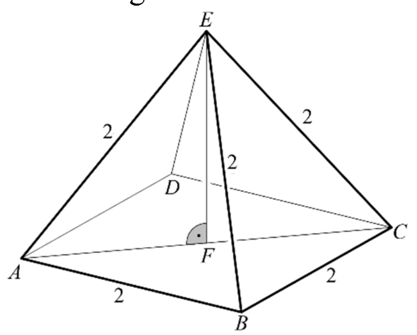
7. a) **Solution 3**		
Number the balls 1 through 30. There are 30! different orders to build the pyramid from the numbered balls. (These are all equally likely, this is the total number of cases.)	1 point	
There are $15 \cdot 14 \cdot 13 \cdot 12$ different ways to place four (numbered) dark balls on the second tier.	1 point	
Use the remaining 26 balls to build the rest of the pyramid in any arbitrary order. This will give $26! \cdot 15 \cdot 14 \cdot 13 \cdot 12$ favourable cases.	1 point	
The probability: $\frac{26!\cdot 15\cdot 14\cdot 13\cdot 12}{30!}=\frac{13}{261}\approx 0.0498$.	1 point	
Total:	**4 points**	

7. a) Solution 4		
Number the 30 slots in the pyramid (from the bottom tier up) 1 through 30 and line up the 30 balls waiting to be placed. Each ball will be placed into the various slots of the pyramid according to its position in the line (the first ball goes to place 1, etc.).	1 point	
There are $\frac{30!}{15!\cdot 15!}$ (= 155 117 520) different ways to line up the 30 balls of different colours. (Total number of cases.)	1 point	
Favourable are the cases in which dark balls will occupy the positions numbered 26, 27, 28 and 29. (The rest of the balls may be placed in any arbitrary order.) The number of favourable cases is therefore: $\frac{26!}{11!\cdot 15!}$ (= 7 726 160).	1 point	
The probability: $\frac{7\,726\,160}{155\,117\,520}=\frac{13}{261}\approx 0.0498.$	1 point	
Total:	**4 points**	

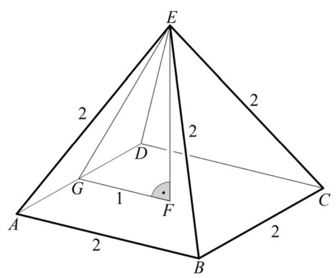
7. a) Solution 5		
(Building the pyramid from the bottom up) there are $\binom{30}{5}$ (= 142 506) different ways to fill the last 5 positions. (Total number of the cases.)	1 point	
In $\binom{15}{5}$ of these cases the last 5 balls will be dark, and in $\binom{15}{1}\cdot\binom{15}{4}$ of these cases there will be 4 dark and 1 light ball. In one fifth of these later cases, that is, in $\frac{15\cdot\binom{15}{4}}{5}$ cases will the last ball be light and the 4 before it dark.	1 point	
The number of favourable cases is: $\binom{15}{5}+3\cdot\binom{15}{4}$ (= 7098).	1 point	
The probability: $\frac{7098}{142\,506}=\frac{13}{261}\approx 0.0498.$	1 point	
Total:	**4 points**	

7. b) **Solution 1**		
(Proof by induction.) The statement is true when $n = 1$, as $1^2 = \frac{1\cdot(1+1)\cdot(2+1)}{6} = 1$.	1 point	
Assuming the statement is true for a certain $k \in \mathbf{N}^+$, we will show that it will also be true for $k + 1$, i.e. $1^2 + 2^2 + ... + k^2 + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$.	1 point	
The induction step: $1^2 + 2^2 + ... + k^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2 =$	1 point	
$= (k+1)\left(\frac{k(2k+1)}{6} + k + 1\right) = (k+1)\cdot\frac{2k^2+k+6k+6}{6} =$ $= \frac{(k+1)(2k^2+7k+6)}{6}$.	1 point	
As $2k^2 + 7k + 6 = (k+2)(2k+3)$,	1 point	
the statement is true for $k + 1$ and so it is also true for any positive integer value.	1 point	
Total:	**6 points**	

7. b) **Solution 2**		
(Proof by induction.) The statement is true when $n = 1$, as $1^2 = \frac{1\cdot(1+1)\cdot(2+1)}{6} = 1$.	1 point	
Assuming the statement is true for a certain $k \in \mathbf{N}^+$, we will show that it will also be true for $k + 1$, i.e. $1^2 + 2^2 + ... + k^2 + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$.	1 point	
Apply the induction step on the left side of the equation: $\frac{k(k+1)(2k+1)}{6} + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$.	1 point	
Divide both sides by $(k + 1)$ where $k \neq -1$, and multiply both sides by 6: $k(2k+1) + 6(k+1) = (k+2)(2k+3)$.	1 point	
$2k^2 + k + 6k + 6 = 2k^2 + 7k + 6$. As this is an identity, it is true for all $k \in \mathbf{N}^+$.	1 point	
All steps taken were equivalent steps, therefore the original statement is also true for all $n \in \mathbf{N}^+$.	1 point	
Total:	**6 points**	

7. c)		
The centres of the four balls in the lower tier (points A, B, C and D in the diagram) and the centre of the single ball in the top tier (point E) form the regular, square-based pyramid $ABCDE$. Each edge of the pyramid is 2 cm long. 	2 points	
The total height of the two-tier pyramid of the balls is (1 + 1 =) 2 cm more than the height (EF) of the above regular pyramid (as the balls in the lower tier touch the base 1 cm below the level of plane $ABCD$, while the highest point of the pyramid is 1 cm above point E).	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The diagonal of square $ABCD$ is $2\sqrt{2}$, half of which is: $AF = \sqrt{2}$ (cm).	1 point*	*The regular pyramid is half of a regular octahedron with 2 cm edges.*
(Apply the Pythagorean theorem) in the right triangle AFE): $EF = \sqrt{2^2 - \sqrt{2}^2} = \sqrt{2}$ (cm).	1 point*	*The solid diagonal of the regular octahedron is the diagonal of a square with 2 cm sides and so the height of the regular pyramid is half of this:* $\sqrt{2}$ cm.
The height of the two-tier pyramid is therefore: ($\sqrt{2} + 2 \approx$) 3.41 cm.	1 point	
Total:	**6 points**	

*Note: The 2 points marked by * may also be given for the following reasoning:*

The height EG of the regular triangle ADE is $\left(\frac{\sqrt{3}}{2} \cdot 2 =\right) \sqrt{3}$ (cm).	1 point	
(Apply the Pythagorean theorem) in the right triangle EGF: $EF = \sqrt{\sqrt{3}^2 - 1^2} = \sqrt{2}$ (cm).	1 point	

8. a)		
(Let S_n be the warehouse stock at the end of the n^{th} month.) After the first month $S_1 = 700 \cdot 0.76 + 60$, after the second month $S_2 = (700 \cdot 0.76 + 60) \cdot 0.76 + 60$, etc. up until after the 18th month when there will be $S_{18} = (...(700 \cdot 0.76 + 60) \cdot 0.76 + ...) \cdot 0.76 + 60$ kg of powdered soup in the warehouse. (There will also be altogether 18 occurrences of both the ·0.76 factors and the +60 terms.)	2 points	
Remove the parentheses and do the operations: $S_{18} = 700 \cdot 0.76^{18} + 60 \cdot 0.76^{17} + 60 \cdot 0.76^{16} + ... + 60 =$	1 point	
$= 700 \cdot 0.76^{18} + 60 \cdot (0.76^{17} + 0.76^{16} + ... + 1)$.	1 point	
The parentheses contain 18 terms of a geometric sequence (the first term is 1, the common ratio is 0.76). Use the formula for the sum of terms of a geometric sequence: $S_{18} = 700 \cdot 0.76^{18} + 60 \cdot \frac{0.76^{18} - 1}{0.76 - 1} \approx$	2 points	
$(\approx 5.009 + 248.211) = 253.22$.	1 point	
Originally, there was a stock of 700 kg in the warehouse and a further ($18 \cdot 60 =$) 1080 kg were added during the 18 months.	1 point	
According to the plans, the total amount of powdered soup sold/given to charity will be approximately (700 + 1080 – 253 =) 1527 kg during the 18 months.	1 point	
Total:	**9 points**	

Note: Award full score if the candidate calculates the rate decrease month by month, rounded reasonably and correctly, and hence gives the correct answer.

month	initial stock (kg)	decrease (kg)
1.	700	168
2.	592	142.1
3.	509.9	122.4
4.	447.5	107.4
5.	400.1	96.0
6.	364.1	87.4
7.	336.7	80.8
8.	315.9	75.8
9.	300.1	72.0

month	initial stock (kg)	decrease (kg)
10.	288.1	69.1
11.	278.9	66.9
12.	272.0	65.3
13.	266.7	64.0
14.	262.7	63.0
15.	259.7	62.3
16.	257.3	61.8
17.	255.6	61.3
18.	254.2	61.0

8. b) **Solution 1**		
(Similar to the steps of part a) there will be $S_n = 700\cdot 0.76^n + 60\cdot 0.76^{n-1} + 60\cdot 0.76^{n-1} + ... + 60$ kilograms of powdered soup after the n^{th} month.	1 point	
Apply the formula for the sum of the geometric sequence: $S_n = 700\cdot 0.76^n + 60\cdot \frac{0.76^n - 1}{0.76 - 1}$.	1 point	
$S_n = 700\cdot 0.76^n - 250\cdot(0.76^n - 1) = 450\cdot 0.76^n + 250$	2 points	
The sequence $\{0.76^n\}$ is a strictly decreasing sequence with a limit of 0.	1 point	
Therefore the sequence $\{S_n\}$ is also strictly decreasing, with a limit of 250. This proves both statements.	2 points	
Total:	**7 points**	

8. b) **Solution 2**		
Apply proof by induction to show that the stock will be more than 250 kg after the n^{th} month, i.e. $S_n > 250$.	1 point	
The statement is true for $n = 1$ ($S_1 = 592$).	1 point	
Assume that the statement is true for a certain $k \in \mathbf{N}^+$, i.e. $S_k > 250$. We will now prove that in this case $S_{k+1} > 250$ is also true.	1 point	
As 76% of the stock remains for the following month, $S_k > 250$ means that more than $(250 \cdot 0.76 =)$ 190 will remain for the following month.	1 point	
This will be increased by the newly produced 60 kg, and so $S_{k+1} > 250$ will, indeed, be true.	1 point	
If the present stock is more than 250 kg then 24% of it is more than $(250 \cdot 0.24 =)$ 60 kg. The stock decreases by more than 60 kg and increases by 60 kg and therefore, altogether, it decreases.	1 point	
As it has already been shown that the stock will be more than 250 kg in each month, it follows now that the stock will keep decreasing every month.	1 point	
Total:	**7 points**	

9. a)		
90 km/h = 1.5 km/min = 1500 m/min	1 point	
The perimeter of the driving wheel is $1740\pi \approx 5466$ (mm), which is approximately 5.47 m.	2 points	
The wheel turns $\frac{1500}{5.47} \approx 274$ times per minute.	1 point	*The RPM of the wheel is* $274\ \frac{1}{\text{min}}$.
Total:	**4 points**	

9. b)		
Travelling at the given average speed the coal consumption is $0.5\cdot 60^2 - 65\cdot 60 + 3800 = 1700$ kg/hour.	1 point	
Therefore the 6100 kilograms of coal will last $6100 : 1700$ (≈ 3.59) hours.	1 point	
During this time the train engine covers a distance of ($3.59\cdot 60 \approx$) 215 kilometres.	1 point	
Total:	**3 points**	

9. c) Solution 1		
If the engine takes $0.5v^2 - 65v + 3800$ kilograms of coal per hour then the 6100 kilograms will last $\frac{6100}{0.5v^2 - 65v + 3800}$ hours.	1 point	
During this time the train engine covers a distance of $\frac{6100v}{0.5v^2 - 65v + 3800}$ kilometres.	1 point	
The function $s(v) = \frac{6100v}{0.5v^2 - 65v + 3800}$ ($50 < v < 100$) may have a maximum wherever its derivative is 0.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
$s'(v) = \frac{6100(0.5v^2 - 65v + 3800) - 6100v(v-65)}{(0.5v^2 - 65v + 3800)^2} =$	2 points	
$= \frac{6100(0.5v^2 - 65v + 3800 - v^2 + 65v)}{(0.5v^2 - 65v + 3800)^2} =$ $= \frac{3050(7600 - v^2)}{(0.5v^2 - 65v + 3800)^2}$	1 point	
$s'(v) = 0$, when $7600 - v^2 = 0$. (As $v > 0$) $v = \sqrt{7600} \approx 87.2$ (which is, in fact, an element of the domain).	1 point	
Values of the derivatives are positive where $v < \sqrt{7600}$, and negative where $v > \sqrt{7600}$. Hence $\sqrt{7600}$ is a global maximum of the function s.	1 point	
The train will travel the furthest on 6.1 tons of coal if it travels at an average speed of about 87 km/h (in which case the distance travelled will be about 275 km).	1 point	
Total:	**9 points**	

Note: $s''(v) = \frac{3050(v^3 - 22\,800v + 988\,000)}{(0.5v^2 - 65v + 3800)^3}$ *and* $s''(\sqrt{7600}) \approx -0.142$; *this means the function s will really have a maximum at the given point.*

9. c) Solution 2		
To travel 1 km at an average speed of v km/h will take the engine $\frac{1}{v}$ hours. (According to the model) the coal consumption on 1 km is: $\frac{1}{v}\cdot(0.5v^2-65v+3800)$ kg.	1 point	
$\frac{1}{v}\cdot(0.5v^2-65v+3800)=0.5v-65+\frac{3800}{v}$	1 point	
The 6.1 tons of coal will last for the longest distance if the consumption on 1 km is minimal.	1 point	
The function $c(v)=0.5v-65+\frac{3800}{v}$ $(50 < v < 100)$ may have a minimum wherever its derivative is 0.	1 point*	*This point is also due if the correct reasoning is reflected only by the solution.*
$c'(v)=0.5-\frac{3800}{v^2}$	2 points*	
$c'(v)=0$, where $v=\sqrt{7600}\approx 87.2$ (as $v > 0$) (which is, in fact, an element of the domain).	1 point*	
Values of the derivative are negative where $v<\sqrt{7600}$, and positive where $v>\sqrt{7600}$. Hence $\sqrt{7600}$ is a global minimum of the function.	1 point*	$c''(v)=\frac{7600}{v^3}$, *the second derivative is positive everywhere, hence* $\sqrt{7600}$ *is a global minimum of c.*
The train will travel the furthest on 6.1 tons of coal if it travels at an average speed of 87 km/h (in which case the distance travelled will be about 275 km).	1 point	
Total:	**9 points**	

*Note: The 5 points marked by * may also be given for the following reasoning:*

(As $v > 0$) the relation between the arithmetic and geometric means will guarantee that $0.5v-65+\frac{3800}{v}\geq 2\cdot\sqrt{0.5v\cdot\frac{3800}{v}}-65=$	1 point	
$=2\cdot\sqrt{1900}-65$ (≈ 22.2).	1 point	*The minimal coal consumption on 1 km is about 22.2 kg.*
The minimal value is assumed where $0.5v=\frac{3800}{v}$.	1 point	
$v^2=7600$, that is $v=\sqrt{7600}$.	1 point	
As $\sqrt{7600}$ (≈ 87.2) is an element of the domain of function c, the function has a global minimum here.	1 point	